Nuclear Spin & Moments

Each nuclear state is assigned a unique "spin" quantum number I, representing the total angular momentum (orbital + intrinsic) of all the nucleons in the nucleus.

$$\vec{I} = \sum_{i=1}^{A} \vec{l}_i + \vec{s}_i$$

$$= \vec{L} + \vec{S} = \sum_{i=1}^{A} \vec{j}_i$$

either of these decompositions can be used (if need be). No dominance bet. LS and jj couplings.

The quantum number $\vec{I}$ satisfies the usual angular momentum relations:

$$|\vec{I}| = \sqrt{I(I+1)}\,\hbar$$

$$I_z = m_I \hbar, \qquad m_I = -I, -I+1, \ldots, I-1, I$$

It may not be apparent why we can neglect the complicated internal structure of the nucleus and treat it as if nucleus were an elementary particle with a single quantum number, representing the intrinsic angular momentum of the "particle". This is possible only because the interactions to which we subject the nucleus (eg. EM fields) are not sufficiently strong to change the internal str. or break the coupling of the nucleons that is responsible for $\vec{I} = \sum_i \vec{l}_i + \vec{s}_i$.

Likewise, for the electronic motion: $\vec{J}^e = \sum_{i=1}^{N} \vec{l}_i^{(e)} + \vec{s}_i^{(e)}$

$\vec{J}^e$ → total electronic angular momentum

Finally, there are cases in which it is most appropriate to deal with the total (nuclear + electronic) angular momentum, usually called $\vec{F}$:

$$\vec{F} = \vec{I} + \vec{J}^e$$

The quantum numbers I and J may be either integral → A = even — nuclear or Z = even — electronic

or half-integral → A = odd — nuclear or Z = odd — electronic

A	Z	I	J	F
Even	Even	Integer +	Integer =	Integer
odd	Even	Half-Integer +	Integer =	Half-int.
Even	Odd	Integer +	Half-Int. =	Half-int.
Odd	Odd	Half-int. +	Half-int. =	Integer

For nuclear ground states, there are several rules for determining spins.

1. All even-Z, even-N nuclei have $I = 0$ (based on pairing) eg. $^{28}_{14}Si_{14}$ has $I = 0$

2. If Z & N are both odd, then $I \in \mathbb{Z} > 0$

e.g. $^{2}_{1}H_{1} \rightarrow I = 1$, $^{10}_{5}B_{5} \rightarrow I = 3$, $^{14}_{7}N_{7} \rightarrow I = 1$

From now on, the ground state nuclear "spin" is simply called "nuclear spin".